Originally appeared in *Pyrotechnics Guild International Bulletin*, Nos. 72 and 73 (1990).

Shimizu Aerial Shell Ballistic Predictions (Part 1)

by K.L. and B.J. Kosanke

Introduction

The effect of varying aerial shell and mortar parameters is a frequent topic of discussion in the display fireworks industry. Dr. Takeo Shimizu has published equations describing both internal (within the mortar) and external (after leaving the mortar) aerial shell ballistics[1]. These equations can be used to make general predictions of the effects of aerial shell and mortar characteristics on shell and mortar performance. Shimizu's work only addressed spherical shells; however, his equations can be used for cylindrical shells providing an appropriate drag coefficient is used. (For the purposes of this article, the drag coefficient of air resistance for cylindrical shells was assumed to be twice the value used by Shimizu for spherical shells.)

In this article, the authors have used the Shimizu equations in order to determine the relative effects of varying aerial shell and mortar characteristics. In the belief that the results generally speak for themselves, the reader is usually left to draw their own conclusions and supply their own rationales. Occasionally, however, this article presents some conclusions or discusses the reasons for the results.

Before presenting the results of this study, two subjects must be presented. The first is a general discussion of the reliability of predictions based on mathematical models (equations). The second is an enumeration of nominal aerial shell and mortar input values used in this study.

Reliability of Predictions Using Mathematical Models

The reliability of predictions made using mathematical models (equations) is almost always limited because simplifications and assumptions usually have been made in their derivation. In some cases, simplifications are made in order to make it possible to perform the calculations; in other cases the simplifications just make it easier or faster to perform the calculations.

As an example of one type of simplification that is required in the case of aerial shell ballistics, consider the following. The microscopic airflow around an aerial shell first being propelled within a mortar and then moving through the air, is so very complex that even the best aerodynamic engineers, using the most sophisticated computers, cannot perform the necessary calculations. In this case, there is no choice except to simplify the calculations by only considering average (macroscopic) effects of airflow. When this is done, it is appropriate to ask whether this limits the accuracy of the calculated results. Of course, the answer is yes; but the errors are not great, and remember, the choice was to simplify the problem or to not perform the calculations at all.

Simplifying assumptions always introduce some error, at least under some circumstances. Thus it is important to consider when such simplifications are appropriate. One such case is when there are uncertainties in input parameters, such as the exact weight, diameter, or amount of lift for a typical shell. Those uncertainties in input parameters cause uncertainties in the results. When those uncertainties in the results are significantly greater than the errors introduced by the simplifying assumptions, the simplifications are appropriate. Another case, when simplifications are appropriate, is when it is only desired to draw general conclusions from the results, and the accuracy of each individual calculated result is of lesser importance. In the present study of aerial shell ballistics, both cases are applicable, and Shimizu's simplifying assumptions are appropriate.

Table 1. Nominal Shell and Mortar Parameters.

Shell Type	Shell Size	Shell Diameter (inches)[a]	Shell Weight (pounds)[a]	Lift Powder Type[c]	Lift Weight (ounces)[a]	Dead Volume (cubic in.)	Mortar Length (inches)
Spherical	3	2.75	0.3	2-3Fg	0.5	12	24
	4	3.70	0.8	2-3Fg	1.0	24	24
	5	4.60	1.5	2-3Fg	1.7	46	30
	6	5.55	2.5	2-3Fg	2.7	72	36
	8	7.50 [b]	5.5	2-3Fg	5.5	150	42
	10	9.50 [b]	11.	2-3Fg	10	290	48
	12	11.50 [b]	18.	2-3Fg	17	520	48
Cylindrical	3	2.75	0.4	2FA	1.0	9	24
	4	3.7	1.0	2FA	1.9	20	24
	5	4.7	2.0	2FA	3.0	35	30
	6	5.7	4.0	2FA	4.5	57	36
	8	7.6	10.	2FA	9.0	121	42
	10	9.5	20.	2FA	16	234	48
	12	11.5	36.	2FA	26	394	48

Notes:

a) Values for spherical shells were derived by interpolating values reported by Shimizu1 p.183.

b) Values derived from Shimizu were 0.05 to 0.1 inches smaller, but it was decided to follow the NFPA guideline that the gap between shell and mortar not exceed 0.5 inches.

c) See Table 3, this suggests that 2Fg powder is the US grade most nearly like the Type 0 lift powder used by Shimizu.

When considering errors introduced by simplifications, there is one more thing that must be addressed. The magnitude of those errors generally depends on how greatly conditions differ between those being calculated and those assumed by the simplification. In effect this introduces limits on when these errors can be safely ignored. As an example of this consider the following. One result predicted by the Shimizu equations is the location of an aerial shell inside a mortar when it will be subjected to the greatest lift pressure. Generally an aerial shell will be 7 to 11 inches above the bottom of the mortar when maximum pressure is reached. As an example of the limits that are imposed by simplifying assumptions, consider the very extreme case of a mortar that is only five-inches tall. In this case, the Shimizu equations still predict that the maximum pressure will occur when the shell has risen 7 to 11 inches in the mortar. Obviously this is impossible! The lesson here is that, while the Shimizu equations may work quite well when using values only a little different from normal, as more and more extreme values are used, one must be more and more cautions in accepting the results.

Within the purpose of this paper, which is only to draw some very general conclusions about internal and external aerial shell ballistics, the authors feel that the errors introduced because of simplifying assumptions are within acceptable limits. However, the reader must be cautioned that no experimental data was collected by the authors for the purpose of verifying the results using the Shimizu equations. Thus, it is not possible to quantify the magnitude of the errors in the results reported here.

Nominal Shell and Mortar Input Values

Table 1 lists the nominal values for input parameters used in this paper. For spherical shells, many of the values were taken from Shimizu[1] by interpolation to US shell sizes. Other values were derived using a combination of measurements of actual shells and mortars, and recommendations of various fireworks experts. Unless otherwise specified, the results reported in this paper use those nominal values as input parameters for the calculations.

Table 2. Comparison between Japanese and American Black Powder Mesh Sizes.

Japanese Powder type[a]	Mesh Range (inches)[b]	American Powder type	Mesh Range (inches)[c]
0	0.016–0.047	4Fg	0.006–0.016
1	0.008–0.016	3Fg	0.012–0.033
2	0.016–0.047	2Fg	0.023–0.047
3	0.047–0.067	4FA	0.033–0.066
4	0.094–0.134	Fg	0.047–0.066
		3FA	0.047–0.079
		2FA	0.066–0.187

Notes:

a) As defined by Shimizu[1], Table 33, p. 170.

b) Values were converted to sieve openings in inches. See Shimizu[1], Table 33, p. 170 for percent passing and retained on sieves.

c) Values derived from information contained in *Engineering Design Handbook* (AMCP 106-175) - Explosives Series - Solid Propellants Part One - The percent passing fine mesh sieve is 3%, and the retained on coarse mesh sieve is 12%.

The Black Powder granulations used in Japan differ from those used in the United States. Table 2 compares Japanese and US granulations. Shimizu reports "characteristic values" for Japanese Black Powder granulations, and these are used as input parameters in his equations. In order to make this paper of greater value to users of US Black Powder granulations, it was necessary to designate which US granulations correspond to Shimizu's characteristic values. These assignments are shown in Table 3.

Table 3. Characteristic Values for Lift Powders

Japanese Powder type	Corresponding American Powder type[b]	Characteristic Values[a]		
		Af ($dm^3/kg \cdot sec$)	AG ($dm^2/kg \cdot sec$)	f/G (dm)
0	2-3Fg	17200	0.256	67100
1	4Fg	17500	0.356	49100
2	2Fg	16000	0.213	75100
3	Fg or 3-4FA	13200	0.182	72500
4	2FA	10900	0.128	85200

Notes:

a) Characteristic values were taken from Shimizu (1), Table 33, p. 170, where A is Charbonnier's "vivacity" of the lift powder in $dm^2/kg \cdot sec$, f is the explosive force of the lift powder in $kg \cdot dm/kg$, and G is the grain shape functions of the lift powder which is dimensionless. (Note dm is decimeter = 10 cm, and kg is kilogram = 1000 grams.)

b) From Table 2, these are the American powder types with mesh range most nearly duplicating those reported by Shimizu.

Table 4. Shell Performance for Nominal Input Parameters.

Shell Type	Shell Size (in.)	Muzzle Velocity (ft/sec)	Maximum Pressure (psi)	Distance to Max. Pres. (inches)	Max. Shell Height (feet)	Time to Max. Ht. (sec.)	Velocity on impact (ft/sec)	Time on Impact (sec)
Spherical	3	358	70	7.5	470	4.4	82	6.5
	4	360	114	7.4	596	5.1	98	7.2
	5	370	127	8.3	680	5.5	107	7.6
	6	389	158	8.4	765	5.9	114	8.0
	8	389	202	9.0	847	6.2	123	8.4
	10	365	248	10.0	893	6.6	133	8.4
	12	278	278	11.4	898	6.7	137	8.3
Cylindrical	3	508	222	5.8	452	4.0	70	6.9
	4	485	271	6.6	551	4.5	81	7.4
	5	479	304	7.1	633	4.9	90	7.8
	6	457	382	7.4	751	5.5	103	8.3
	8	432	515	7.8	878	6.2	119	8.7
	10	400	610	8.7	939	6.6	131	8.8
	12	358	721	9.5	929	6.8	139	8.5

Nominal Aerial Shell Performance Values

Table 4 lists the shell performance values predicted by the Shimizu equations, when using the nominal input values given in Table 1. Figures 1 through 3 present maximum mortar pressure, maximum shell height, and time to maximum shell height as functions of shell size for both spherical and cylindrical shells. It is of interest to note that maximum mortar pressures for cylindrical shells are approximately 2.5 times greater than those for spherical shells. Of course, the importance of this result is that cylindrical shells place considerably more stress on a mortar than do spherical shells, a fact well known to experienced pyrotechnicians.

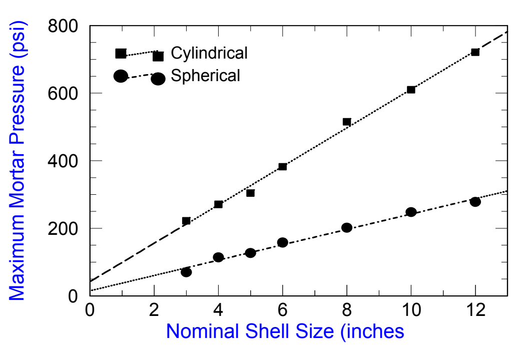

Figure 1. Maximum mortar pressure as a function of shell size for nominal input parameters.

Figure 2 also includes empirically determined burst heights for spherical shells[2]. This curve represents a rather limited amount of data; however, it is in general agreement with some data published by Shimizu[1]. This experimentally determined data was included because it was felt it must be acknowledged that the data for large shells deviate from the maximum shell heights predicted using the Shimizu equations. (At the time of this writing, the reason for this difference has not been established.)

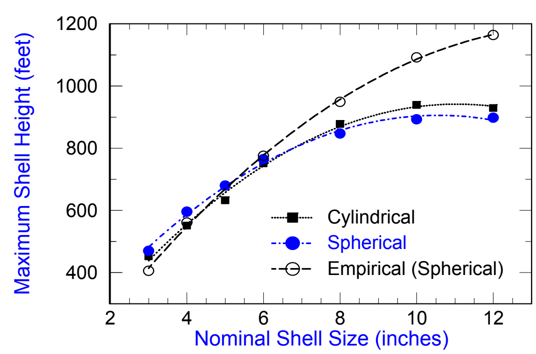

Figure 2. Maximum shell height as a function of shell size for nominal input parameters.

In addition, it may be of interest to note that:

- Muzzle velocities are largely independent of shell size.

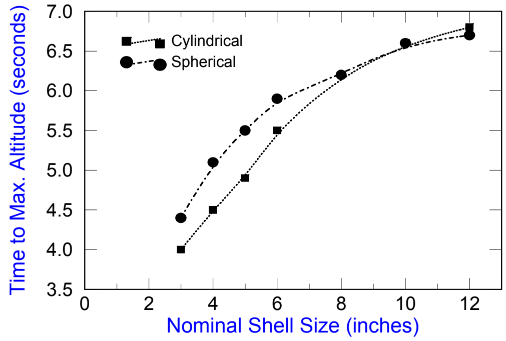

Figure 3. Time taken to reach maximum shell height as a function of shell size for nominal input parameters.

- Maximum mortar pressures are reached before the shells rise very far above the bottom of the mortar.
- Rise times for shells are shorter than fall times.

Readers are again cautioned to consider these shell performance values only within the context of this paper. These values are calculated results based on numerous assumptions and only for the nominal input values assumed. These performance values are not the results of actual measurements and they may be only approximately correct.

Effects of Mortar Length

Over the years, there has probably been more speculation regarding the effect of mortar length on the flight of aerial shells than any other single factor. The results of calculations of maximum shell height for 3, 6, and 12-inch shells as a function of mortar length are listed in Table 5. Maximum shell heights are listed both in absolute terms and as a percent of the heights achieved when using mortar lengths 20 times the diameter. These same data are presented in Figure 4. For convenience in plotting, mortar lengths are expressed as multiples of mortar internal diameters.

It may be of interest to examine the data in order to evaluate the appropriateness of the rule-of-thumb recommending use of mortars 5-times their ID for shells less than 8-inches and 4-times their ID for shells 8-inches or more. For small shells, it seems there might be an advantage in using mortars that were somewhat longer. However, given the burst radii of hard breaking shells[3], it does not seem that the 5-times diameter rule represents a safety concern.

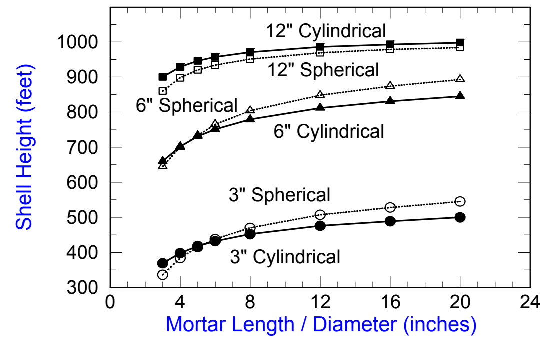

Figure 4. Maximum shell height as a function of mortar length.

It might also be of interest to comment on the relationship between a shell's muzzle velocity and the maximum height it attains. In order to do this, consider the muzzle velocity and maximum height data for three-inch spherical shells, listed in Table 6 and shown in Figure 5. With increasing mortar length, muzzle velocity and maximum height both increase; however, the increase in maximum height is not as great as the increase in muzzle velocity. The reason for this difference is that aerodynamic drag is a function of a shell's velocity[1,4]. The faster a shell is moving, the greater are the losses due to drag forces. Thus increases in muzzle velocity cause greater drag forces, which in turn allow less than proportional increases in shell height.

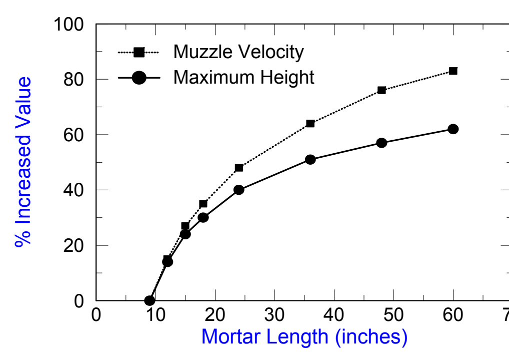

Figure 5. Comparison of increases in maximum shell height and muzzle velocity as a function of mortar length for three-inch spherical shells.

End of Part 1: (The remainder of this article will continue to address the effects of altering input values; for example varying mortar to shell clearance, shell weights, and loading spaces.)

Table 5. Effect of Mortar Length on Maximum Shell Height.

	Mortar Length Divided by Diameter	3" Shell		6" Shell		12" Shell	
		Height (feet)	Percent (a)	Height (feet)	Percent (a)	Height (feet)	Percent (a)
Spherical	3	336	62	645	72	860	87
Spherical	4	384	70	701	78	898	91
Spherical	5	415	76	733	82	920	93
Spherical	6	438	80	765	86	934	95
Spherical	8	470	86	804	90	951	97
Spherical	12	507	93	848	95	969	98
Spherical	16	528	97	874	98	979	99
Spherical	20	545	100	893	100	984	100
Cylindrical	3	369	74	660	78	900	90
Cylindrical	4	398	80	702	83	929	93
Cylindrical	5	418	84	731	87	946	95
Cylindrical	6	432	86	751	89	957	96
Cylindrical	8	452	90	779	92	971	97
Cylindrical	12	476	95	812	96	986	99
Cylindrical	16	489	98	831	98	993	99
Cylindrical	20	500	100	845	100	998	100

(a) Height expressed as the percent of the height reached when mortar length is 20 times the mortar diameter.

Table 6. Effect of Mortar Length on Maximum Shell Height and Muzzle Velocity for Three-Inch Spherical Shells.

Mortar Length (inches)	Muzzle Velocity (ft/sec)	Percent Increased Velocity (a)	Maximum Height (feet)	Percent Increased Shell Height (a)
9	242	0	336	0
12	279	15	383	14
15	307	27	415	24
18	327	35	438	30
24	358	48	470	40
36	398	64	507	51
48	425	76	529	57
60	444	83	545	62

(a) Muzzle velocity and height as the percent increase to that for a nine-inch long mortar.

References for Part 1

1) Shimizu, T., 1985. *Fireworks from a Physical Standpoint Part III*, Pyrotechnica Publications, Austin, TX.

2) Kosanke, K.L., Schwertly, L.A., and Kosanke, B.J., "Report of Aerial Shell Burst Height Measurements", *PGI Bulletin*, No. 68 (1990).

3) Kosanke, K.L. and B.J., "Japanese Shell Break Radii", *PGI Bulletin*, No. 59 (1988).

4) Kosanke, K.L. and B.J., "Computer Modeling of Aerial Shell Ballistics," *Pyrotechnica XIV* (1992).

Originally appeared in *Pyrotechnics Guild International Bulletin*, No. 73 (1990).

Shimizu Aerial Shell Ballistic Predictions (Part 2)

by K.L. and B.J. Kosanke

(Continuation of Part 1, which appeared in *Pyrotechnics Guild International Bulletin* No. 72 (1990).

Effects of Shell Clearance in Mortar

Another area of frequent speculation is the effect of various shell clearances within mortars. However, Shimizu warns that his Black Powder characteristic values are only correct for shells with diameters about 11 percent smaller than the mortar. (In effect, this is one of the simplifying assumptions he has made.) It is not possible to run calculations of the effect of varying shell clearance, without having the appropriate Black Powder characteristic values. Unfortunately, the derivation of the needed values is beyond the present limits of the authors' expertise, and thus the desired clearance calculations cannot be performed and reported here.

Effects of Shell Lift Weight

The results of calculations of maximum shell height and maximum mortar pressure as functions of the amount of lift charge are listed in Table 7 and shown in Figures 6 and 7. For these calculations, the range of values used for lift charge weights was limited to 80 percent through 140 percent of the nominal amounts listed in Table 1 (Part 1). Even though results for more extreme values would certainly be of interest, these are not reported here. This is because there was evidence that the characteristic values for Black Powder were not appropriate for use in more extreme cases. Rather than include highly suspicious results, the authors chose the conservative approach of limiting the range of reported results.

As can be seen when comparing Figures 6 and 7, the effect of lift charge amount on maximum shell height, is predicted to be much less than its effect on maximum mortar pressure. For example, varying lift charge weight for 6-inch spherical shells produced an increase in maximum shell height of 34 percent; it simultaneously produced an increase in maximum mortar pressure of 164 percent!

Table 7. Effect of Shell Lift Weight on Maximum Shell Height and Maximum Mortar Pressure.

3" Spherical			3" Cylindrical			6" Spherical		
Lift Weight (ounces)	Max. Height (feet)	Max. Pressure (psi)	Lift Weight (ounces)	Max. Height (feet)	Max. Pressure (psi)	Lift Weight (ounces)	Max. Height (feet)	Max. Pressure (psi)
0.4	401	44	0.8	401	137	2.2	672	107
0.5	470	70	1.0	452	222	2.7	765	158
0.6	524	102	1.2	492	330	3.2	841	217
0.7	571	140	1.4	525	463	3.7	901	283
6" Cylindrical			**12" Spherical**			**12" Cylindrical**		
Lift Weight (ounces)	Max. Height (feet)	Max. Pressure (psi)	Lift Weight (ounces)	Max. Height (feet)	Max. Pressure (psi)	Lift Weight (ounces)	Max. Height (feet)	Max. Pressure (psi)
3.6	661	246	14	827	204	21	853	506
4.5	751	382	17	898	278	26	929	726
5.4	817	545	20	944	358	31	975	956
6.3	869	735	23	975	443	36	1006	1209

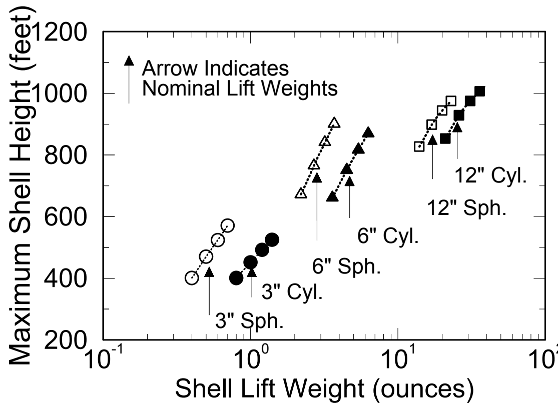

Figure 6. Maximum shell height as a function of lift weight.

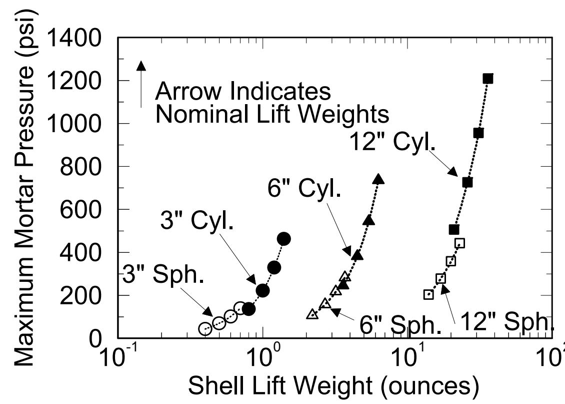

Figure 7. Maximum mortar pressure as a function of shell lift weight.

Effects of Lift Charge Type

The results of calculations of maximum shell height, maximum mortar pressure, and distance to maximum pressure as functions of lift charge type are listed in Table 8. The authors have some concern as to whether the results are totally believable (e.g. maximum mortar pressures for 4Fg lift powder are consistently less than expected when compared to reported values for other granulations). In part, this may be a result of the authors' assigning US Black Powder granulations to characteristic values for Japanese lift powder. Nonetheless, several things seem clear:

- For small and medium spherical shells, the use of finer grained powders is predicted to be useful in propelling the shells to their proper heights.
- For large spherical shells and all cylindrical shells, the use of coarser grained powders is preferred because the use of finer grained powders produces little if any gain in maximum shell height, while at the same time producing much higher mortar pressures.
- The use of progressively finer lift powders has the expected effect of decreasing the distance travelled by the shell in the mortar before maximum mortar pressure is reached.

The results of calculations of maximum shell height and maximum mortar pressure as functions of shell weight are listed in Table 9 and shown in Figures 8 and 9. The results for maximum shell height, at first seem somewhat surprising. The calculations suggest that small spherical shells will travel to greater heights when they are made heavier. The reason is that for each shell size and

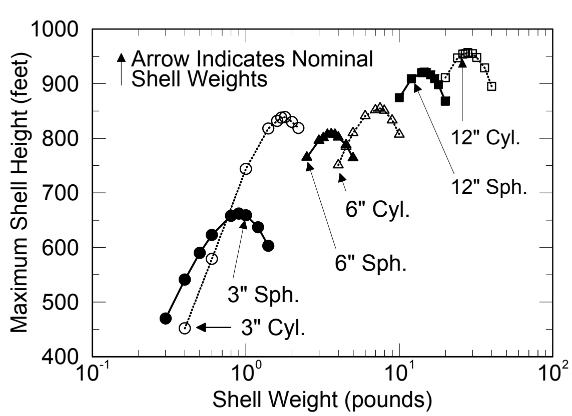

Figure 8. Maximum shell height as a function of shell weight.

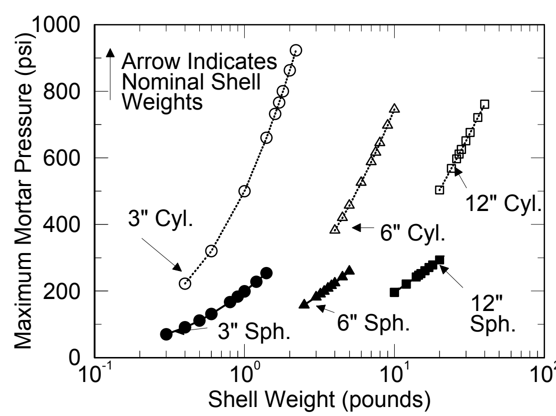

Figure 9. Maximum mortar pressure as a function of shell weight.

Table 8. Effect of Shell Lift Type on Maximum Shell Height, Maximum Mortar Pressure and Distance of Shell Travel within Mortar at the Moment of Maximum Pressure.

	3" Spherical			3" Cylindrical		
Lift Type	Max. Height (feet)	Max. Pressure (psi)	Dist. to Max. press. (inches)	Max. Height (feet)	Max. Pressure (psi)	Dist. to Max. Press. (inches)
2FA	346	29	9.5	452	222	5.8
3-4FA	399	42	8.7	477	317	5.0
2Fg	455	61	8.0	503	449	4.4
2-3Fg	470	70	7.5	501	500	4.0
4Fg	457	71	6.75	468	479	3.6
	6" Spherical			6" Cylindrical		
Lift Type	Max. Height (feet)	Max. Pressure (psi)	Dist. to Max. press. (inches)	Max. Height (feet)	Max. Pressure (psi)	Dist. to Max. Press. (inches)
2FA	625	70	12.1	751	382	7.4
3-4FA	694	100	10.4	737	500	6.2
2Fg	768	142	9.2	736	657	5.6
2-3Fg	765	158	8.4	681	683	5.2
4Fg	687	151	7.4	546	594	4.8
	12" Spherical			12" Cylindrical		
Lift Type	Max. Height (feet)	Max. Pressure (psi)	Dist. to Max. press. (inches)	Max. Height (feet)	Max. Pressure (psi)	Dist. to Max. Press. (inches)
2FA	964	158	16.5	929	721	9.5
3-4FA	964	206	13.7	706	790	8.5
2Fg	982	269	12.3	600	923	8.1
2-3Fg	898	278	11.4	475	886	7.9
4Fg	681	241	10.5	292	698	7.6

lift charge weight, there is an optimum shell weight that results in the greatest height for the shell. At lesser weights the situation becomes increasingly like a person trying to throw a feather; it is almost impossible to throw a feather farther than a few feet no matter how hard it is thrown. At weights greater than the optimum, the situation becomes increasingly like a person trying to throw a cement block; again, it is almost impossible to throw a cement block more than a short distance. However, for objects near the optimum size and weight (e.g. a baseball), it is relatively easy for a person to throw the object a hundred feet or more. Following this analogy, small shells fall more nearly into the category of feathers rather than cement blocks, and an increase in their weight actually causes them to be propelled to greater heights. It may be of interest to note that nominal 8 and 10-inch spherical shells, and 12-inch cylindrical shells are very nearly at their optimum projection weights.

As would be expected, maximum mortar pressure universally increases as shell weights increase. This is primarily the result of the shell's increasing inertia. Heavier shells accelerate more slowly in response to a given lift gas pressure. Accordingly, heavier shells spend a longer time traveling any given distance within the mortar. In turn, this means that during the early stages of the shell's travel within the mortar, greater percentages of the lift power will have been consumed, generating more gas in the same space, which manifests itself as greater mortar pressure.

It may be of interest to note that, independent of shell size, all shells, of approximately the optimum shell weight, result in nearly constant maximum mortar pressures. For spherical shells this is roughly 200 psi, and for cylindrical shells this is roughly 700 psi.

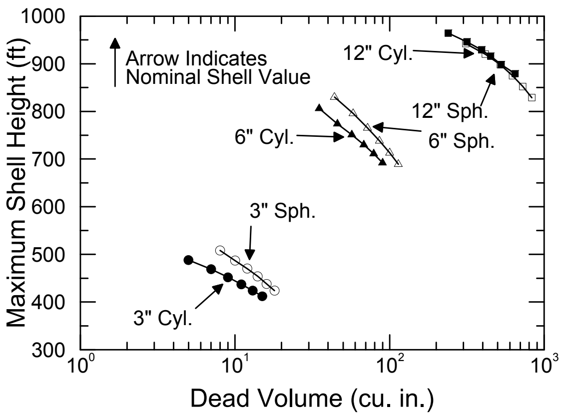

Figure 10. Maximum shell height as a function of shell dead volume.

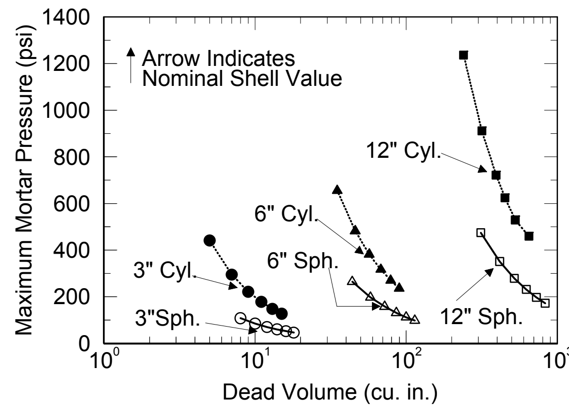

Figure 11. Maximum mortar pressure as a function of dead volume.

Effects of Dead Volume

Dead volume (also called loading space) is defined as the unoccupied volume below a shell in a mortar. Results of calculations of maximum shell heights and maximum mortar pressures as functions of dead volume are listed in Table 10 and shown in Figures 10 and 11. It should be noted

Table 9. Effect of Shell Weight on Maximum Shell Height and Maximum Mortar Pressure.

3" Cylindrical			3" Spherical			6" Cylindrical		
Shell Weight (lbs.)	Max. Height (feet)	Max. Pressure (psi)	Shell Weight (lbs.)	Max. Height (feet)	Max. Pressure (psi)	Shell Weight (lbs.)	Max. Height (feet)	Max. Pressure (psi)
0.3	470	70	0.4	452	222	2.5	765	158
0.4	541	91	0.6	579	320	3.0	796	182
0.5	590	111	1.0	744	500	3.2	801	191
0.6	623	131	1.4	818	660	3.4	807	199
0.8	658	167	1.6	832	732	3.6	808	208
0.9	662	183	1.7	838	766	3.8	807	216
1.0	659	199	1.8	839	800	4.0	802	224
1.2	637	228	2.0	830	863	4.5	788	242
1.4	603	254	2.2	819	923	5.0	764	259

6" Cylindrical			12" Spherical			12" Cylindrical		
Shell Weight (lbs.)	Max. Height (feet)	Max. Pressure (psi)	Shell Weight (lbs.)	Max. Height (feet)	Max. Pressure (psi)	Shell Weight (lbs.)	Max. Height (feet)	Max. Pressure (psi)
4.0	751	382	10.	874	196	20.	911	503
4.5	785	420	12.	909	221	24.	947	568
5.0	810	457	14.	920	242	26.	954	597
6.0	841	526	14.5	921	247	27.	955	611
7.0	852	588	15.	920	252	28.	957	625
7.5	855	617	16.	916	261	30.	955	651
8.0	851	645	17.	909	270	32.	948	676
9.0	833	698	18.	898	278	36.	929	721
10.0	807	746	20.	868	293	40.	895	761

Table 10. Effect of Dead Volume on Maximum Shell Height and Maximum Mortar Pressure.

3" Spherical			3" Spherical			6" Spherical		
Dead Volume (cu. in.)	Max. Height (feet)	Max. Pressure (psi)	Dead Volume (cu. in.)	Max. Height (feet)	Max. Pressure (psi)	Dead Volume (cu. in.)	Max. Height (feet)	Max. Pressure (psi)
8	508	108	5	488	441	44	830	264
10	487	85	7	469	295	58	796	197
12	470	70	9	452	221	72	765	158
14	454	60	11	437	178	86	738	131
16	438	52	13	424	148	100	713	112
18	424	46	15	412	127	114	689	98
6" Spherical			**12" Spherical**			**12" Spherical**		
Dead Volume (cu. in.)	Max. Height (feet)	Max. Pressure (psi)	Dead Volume (cu. in.)	Max. Height (feet)	Max. Pressure (psi)	Dead Volume (cu. in.)	Max. Height (feet)	Max. Pressure (psi)
35	806	655	312	942	474	240	964	1236
46	774	482	416	920	351	317	946	911
57	751	382	520	898	278	394	929	721
68	730	316	624	875	231	451	916	624
79	711	269	728	852	197	528	898	529
90	692	235	832	829	172	650	879	459

that while dead volume is predicted to have an effect on maximum shell height, the effect is not particularly great. For example, a 60% increase in dead volume for a 6-inch cylindrical shell results in only a 7% decrease in maximum shell height. (Note that a 60% increase in dead volume is equivalent to raising the shell an extra 1¼-inch off the bottom of the mortar.) One could conclude from this observation that small amounts of debris, remaining in mortars between firings, and thereby increasing dead volume, will not result in an unsafe decrease in maximum shell height. This is one reason (combined with personnel safety considerations) why it is no longer recommended that mortars be cleaned after each use during a manually fired display. The effect of dead volume on maximum mortar pressure, shown in Figure 11, is of much greater consequence. For example a 40% decrease in dead volume for a 6-inch cylindrical shell results in a 71% increase in maximum mortar pressure (Note that a 40% reduction in dead volume is equivalent to pushing the shell ¾-inch further into the mortar.). Thus, when attempting to fire a massive shell and have both the shell and mortar survive the process, one should employ ample dead volume. In many cases, the modest loss in shell height that results can be eliminated by using a slightly longer mortar. As an alternative, even if slightly more lift is used to fully restore the shell's height, maximum mortar pressures will still be lower than was the case when there was less dead volume.

Dead volume is also one reason why spherical shells, even heavy ones, can be lifted using rather fine-grained powders. The shape of spherical shells automatically provides ample dead volume, which tends to reduce the maximum mortar pressures below that which would normally result from the use of fine grained (faster burning) lift powder.

Conclusion

The information presented in this article is only intended to illustrate the general effects of varying shell and mortar parameters. It is not intended to imply that any of the results can be taken as precisely accurate. In spite of the limitations implicit in these data, they should prove to be of interest to both manufacturers and display companies.

Acknowledgments

The authors wish to gratefully acknowledge T. Shimizu for his original efforts in developing the formulas used in these calculations and for reviewing an earlier draft of this article. The authors wish to acknowledge D. Rowe for assistance with defining nominal input parameters for large cylindrical shells. Finally, the authors wish to acknowledge R. Winokur and J. Bergman for their assistance in editing this article.